

Aspects of symmetry breaking in $SO(10)$ GUTs

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Abstract. I review some recent results on the Higgs sector of minimal $SO(10)$ grand unified theories both with and without supersymmetry. It is shown that nonsupersymmetric $SO(10)$ with just one adjoint triggering the first stage of the symmetry breaking does provide a successful gauge unification when radiative corrections are taken into account in the scalar potential, while in the supersymmetric case it is argued that the troubles in achieving a phenomenologically viable breaking with representations up to the adjoint are overcome by considering the flipped $SO(10)$ embedding of the hypercharge.

1. Introduction

While the standard model (SM) matter quantum numbers nicely fit in a few lowest-dimensional representations of the unified groups such as $SU(5)$ or $SO(10)$, this synthetic process has no counterpart in the Higgs sector where a larger set of higher-dimensional representations is usually needed in order to spontaneously break the enhanced gauge symmetry down to the SM gauge group. In this respect, establishing the minimal Higgs content needed for the grand unified theory (GUT) breaking is a basic question which has been already addressed in the early days of the GUT program. Let us remark that the quest for the simplest Higgs sector is driven not only by aesthetic criteria but it is also a phenomenologically relevant issue related to tractability and predictivity of the models. Indeed, the details of the symmetry breaking pattern, sometimes overlooked in the phenomenological analysis, give further constraints on the low-energy observables such as the proton decay and the effective SM flavor structure.

Here we will focus mainly on $SO(10)$ GUTs, both with and without supersymmetry (SUSY). Let us recall that before considering any symmetry breaking dynamics the following representations are required by the group theory in order to achieve a full breaking of $SO(10)$ to the SM:

- (i) 16_H or 126_H for rank reduction. Their SM-singlet vacuum expectation value (VEV) communicates the $B - L$ breaking to neutrino masses, but preserves an $SU(5)$ little group.
- (ii) 45_H or 54_H or 210_H for the further $SU(5)$ breaking. They admit for little groups different from $SU(5) \otimes U(1)$, yielding the SM when intersected with the $SU(5)$ remnant of (i).

It should be also mentioned that a one-step $SO(10) \rightarrow$ SM breaking can be achieved via only one 144_H irreducible Higgs representation [1]. However, such a setting requires an extended matter sector, including 45_F and 120_F multiplets, in order to accommodate realistic fermion masses [2].

¹ In collaboration with Stefano Bertolini (INFN and SISSA, Trieste) and Michal Malinský (IFIC, Valencia). Talk given at DISCRETE'10 - *Symposium on Prospects in the Physics of Discrete Symmetries*, 6-11 Dec 2010, Rome.

While the choice between 16_H or 126_H is a model dependent issue related to the details of the Yukawa sector, the simplest option in the list (ii) is certainly given by the adjoint 45_H . However, since the early 1980's, it has been observed that the vacuum dynamics aligns the adjoint along an $SU(5) \otimes U(1)$ direction, making the choice of the 45_H alone not phenomenologically viable. In the nonsupersymmetric case the alignment is only approximate [3, 4, 5], but it is such to clash with unification constraints which do not allow for any $SU(5)$ -like intermediate stage, while in the supersymmetric limit the alignment is exact due to F-flatness [6, 7, 8], thus never landing to a supersymmetric SM vacuum. In the next sections we will review all these issues in more detail and provide a way out to the aforementioned problem of the vacuum alignment.

2. Adjoint breaking in nonsupersymmetric $SO(10)$

Let us consider the most general renormalizable tree level scalar potential which can be constructed out of 45_H and 16_H in nonsupersymmetric $SO(10)$

$$V_0 = V_{45_H} + V_{16_H} + V_{45_H 16_H}, \quad (1)$$

where, according to the notation in Ref. [9]

$$V_{45_H} = -\frac{\mu^2}{2} \text{Tr } 45_H^2 + \frac{a_1}{4} (\text{Tr } 45_H^2)^2 + \frac{a_2}{4} \text{Tr } 45_H^4, \quad (2)$$

$$V_{16_H} = -\frac{\nu^2}{2} 16_H^\dagger 16_H + \frac{\lambda_1}{4} (16_H^\dagger 16_H)^2 + \frac{\lambda_2}{4} (16_H^T \Gamma 16_H)(16_H^\dagger \Gamma 16_H^*), \quad (3)$$

$$V_{45_H 16_H} = \alpha (16_H^\dagger 16_H) \text{Tr } 45_H^2 + \beta 16_H^\dagger 45_H^2 16_H + \tau 16_H^\dagger 45_H 16_H. \quad (4)$$

There are in general three SM-singlets in the $45_H \oplus 16_H$ reducible representation of $SO(10)$. Their VEVs are defined in the following way

$$\omega_{B-L} \subset \langle (15, 1, 1) \rangle \subset \langle 45_H \rangle, \quad \omega_R \subset \langle (1, 1, 3) \rangle \subset \langle 45_H \rangle, \quad \chi_R \subset \langle (\bar{4}, 1, 2) \rangle \subset \langle 16_H \rangle, \quad (5)$$

where the three submultiplets above are labeled according to the $SO(10)$ subalgebra $4_C 2_L 2_R$ (shorthand notation for $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$). Different VEV configurations trigger the spontaneous breakdown of the $SO(10)$ symmetry into the following subgroups. Using a self-explanatory notation, for $\chi_R = 0$ one finds

$$\begin{aligned} \omega_R = 0, \omega_{B-L} \neq 0 : & \quad 3_c 2_L 2_R 1_{B-L}, \\ \omega_R \neq 0, \omega_{B-L} = 0 : & \quad 4_C 2_L 1_R, \\ \omega_R \neq 0, \omega_{B-L} \neq 0 : & \quad 3_c 2_L 1_R 1_{B-L}, \\ \omega_R = -\omega_{B-L} \neq 0 : & \quad \text{flipped } SU(5) \otimes U(1)_Z, \\ \omega_R = \omega_{B-L} \neq 0 : & \quad \text{standard } SU(5) \otimes U(1)_Z. \end{aligned} \quad (6)$$

When $\chi_R \neq 0$ all intermediate symmetries are spontaneously broken down to the $3_c 2_L 1_Y$ of the SM with the exception of the last case which leaves the standard $SU(5)$ unbroken. In this language, the potentially viable breaking chains fulfilling the basic gauge unification constraints (labeled as VIII and XII in Ref. [10]) correspond to the settings with:

$$\begin{aligned} \omega_{B-L} \gg \omega_R > \chi_R : SO(10) \rightarrow 3_c 2_L 2_R 1_{B-L} \rightarrow 3_c 2_L 1_R 1_{B-L} \rightarrow 3_c 2_L 1_Y & \quad \text{Chain VIII,} \\ \omega_R \gg \omega_{B-L} > \chi_R : SO(10) \rightarrow 4_C 2_L 1_R \rightarrow 3_c 2_L 1_R 1_{B-L} \rightarrow 3_c 2_L 1_Y & \quad \text{Chain XII.} \end{aligned}$$

The parameters (couplings and VEVs) of the scalar potential are constrained by the requirements of boundedness and the absence of tachyonic states, ensuring that the vacuum is stable and the

stationary points correspond to physical minima. In particular, from the shape of the tree level masses of the $(8, 1, 0)$ and $(1, 3, 0)$ SM sub-multiplets of 45_H :

$$M^2(1, 3, 0) = 2a_2(\omega_{B-L} - \omega_R)(\omega_{B-L} + 2\omega_R), \quad (7)$$

$$M^2(8, 1, 0) = 2a_2(\omega_R - \omega_{B-L})(\omega_R + 2\omega_{B-L}), \quad (8)$$

which can not be simultaneously positive unless

$$a_2 < 0, \quad -2 < \omega_{B-L}/\omega_R < -\frac{1}{2}, \quad (9)$$

one concludes that the only vacuum configurations allowed are those in the close vicinity of the flipped $SU(5) \otimes U(1)_Z$ setting. Hence, the large hierarchy (of about four orders of magnitude) between ω_R and ω_{B-L} , required by gauge coupling unification (cf. chains VIII and XII in Ref. [10]) cannot be achieved. This is the key point of the classical argument that the nonsupersymmetric $SO(10)$ GUTs with only one adjoint responsible for the first stage of the $SO(10)$ breakdown can not support the phenomenologically favoured symmetry breaking chains.

2.1. A tree level accident

The rationale for understanding the strong correlation among the masses of the states $(1, 3, 0)$ and $(8, 1, 0)$ can be obtained by looking at the enhancement of the global symmetry in a trivial limit of the scalar potential. When only trivial invariants of both 45_H and 16_H are considered ($a_2 = \lambda_2 = \beta = \tau = 0$) the global symmetry of V_0 is $O(45) \otimes O(32)$. This is then broken spontaneously into $O(44) \otimes O(31)$ by the 45_H and 16_H VEVs yielding $44+31=75$ Goldstone bosons (GB) in the scalar spectrum. The gauge $SO(10)$ symmetry is at the same time broken down to the SM gauge group. Therefore $75-33=42$ pseudo-Goldstone bosons (PGB) are left in the spectrum and their masses should generally receive contributions from all the explicitly breaking terms a_2 , λ_2 , β and τ . Since the states $(1, 3, 0)$ and $(8, 1, 0)$ belong to this set of PGB, generally one would expect their mass to depend on all of a_2 , λ_2 , β , τ parameters. The absence of mass contributions proportional to λ_2 , β , τ is just an easily understood accident of the tree level potential [9], but nothing prevents those couplings from contributing to the PGB masses at the quantum level.

2.2. The quantum vacuum

The relevant one-loop correction to the $(1, 3, 0)$ and $(8, 1, 0)$ PGB masses can be conveniently computed by means of the one-loop effective potential (EP). The one-loop EP can be formally written as

$$V_{\text{eff}} = V_0 + \Delta V_s + \Delta V_f + \Delta V_g, \quad (10)$$

where $\Delta V_{s,f,g}$ denote the contributions from scalars, fermions and gauge bosons respectively. In dimensional regularization with modified minimal subtraction ($\overline{\text{MS}}$) and in the Landau gauge, they are given by

$$\Delta V_i(45_H, 16_H, \mu) = \frac{\alpha_i}{64\pi^2} \text{Tr} \left[\mathcal{M}_i^4(45_H, 16_H) \left(\log \frac{\mathcal{M}_i^2(45_H, 16_H)}{\mu^2} - \beta_i \right) \right], \quad (11)$$

where $(\alpha_s, \alpha_f, \alpha_g) = (1, -2, 3)$, $(\beta_s, \beta_f, \beta_g) = (\frac{3}{2}, \frac{3}{2}, \frac{5}{6})$ and \mathcal{M}_s , \mathcal{M}_f and \mathcal{M}_g are the functional scalar, fermion and gauge boson mass matrices respectively, as obtained from the tree level potential. In the following we will neglect the fermionic component of the EP since there are no fermionic states at the unification scale M_G . The running masses \overline{m}_{ab}^2 are defined by

$$\overline{m}_{ab}^2 \equiv \frac{\partial^2 V_{\text{eff}}(45_H, 16_H, \mu)}{\partial \psi_a \partial \psi_b} \Big|_{\langle \psi \rangle}, \quad (12)$$

where ψ_a and ψ_b are generic scalar field components and the VEVs (denoted collectively $\langle\psi\rangle$) obey the one-loop stationary equations. For a given eigenvalue the physical (pole) masses M_a^2 are then obtained by

$$M_a^2 = \overline{m}_a^2 + \Delta\Sigma_a(M_a^2), \quad (13)$$

where $\Delta\Sigma_a(p^2) = \Sigma_a(p^2) - \Sigma_a(0)$ and Σ_a are the $\overline{\text{MS}}$ renormalized self-energies. Of particular interest is the case when M_a is substantially smaller than the mass (M_G) of the particles that contribute to Σ_a . At $\mu = M_G$ in the limit $M_a^2 \ll M_G^2$ one has

$$\Delta\Sigma_a(M_a^2) = O(M_a^4/M_G^2). \quad (14)$$

In this case the running mass computed from Eq. (12) contains the leading gauge independent corrections.

The stringent tree level constraint on the ratio ω_{B-L}/ω_R , coming from the positivity of the masses of the states $(1, 3, 0)$ and $(8, 1, 0)$, that forbids non- $SU(5)$ vacua, follows from the fact that the masses depend only on the parameter a_2 . On the other hand, from the discussion of the would-be global symmetries of the scalar potential we should in general expect their masses to depend on other terms in the scalar potential (in particular β , τ and gauge interactions at the one-loop level). The calculation of the EP running mass from Eq. (12) leads for the states $(1, 3, 0)$ and $(8, 1, 0)$ at $\mu = M_G$ to the mass shifts

$$\Delta M^2(1, 3, 0) = \frac{1}{4\pi^2} [\tau^2 + \beta^2(2\omega_R^2 - \omega_R\omega_{B-L} + 2\omega_{B-L}^2) + g^4(16\omega_R^2 + \omega_R\omega_{B-L} + 19\omega_{B-L}^2)] \quad (15)$$

$$\Delta M^2(8, 1, 0) = \frac{1}{4\pi^2} [\tau^2 + \beta^2(\omega_R^2 - \omega_R\omega_{B-L} + 3\omega_{B-L}^2) + g^4(13\omega_R^2 + \omega_R\omega_{B-L} + 22\omega_{B-L}^2)] \quad (16)$$

where the subleading (and gauge dependent) logarithmic terms are neglected and we have taken for simplicity $\chi_R = 0$, given that $\chi_R \ll \omega_{R,B-L}$ by unification constraints. For more details we refer the reader to Refs. [9, 11]. By comparing Eqs. (15)–(16) with Eqs. (7)–(8) it is clear that a consistent scalar mass spectrum can be obtained for the non- $SU(5)$ vacua, at variance with the tree level result. In particular, a hierarchy between ω_{B-L} and ω_R (as required by unification), while keeping the scalar states positive (minimum condition), is now possible just by taking $|a_2| \lesssim 10^{-2}$. This corresponds to keeping the masses of the PGB $(1, 3, 0)$ and $(8, 1, 0)$ one order of magnitude below M_G , which makes the EP potential computation self-consistent (since the self-energies in Eq. (13) can be neglected) and has the welcome effect of raising a little M_G . Let us also stress that this result is inherent to all the nonsupersymmetric $SO(10)$ models with one adjoint 45_H triggering the first stage of the GUT breaking: just one additional GUT representation interacting with the adjoint is needed in order to open up the non- $SU(5)$ vacua at the quantum level.

Nevertheless, the simplest scenario featuring the Higgs scalars in $10_H \oplus 16_H \oplus 45_H$ of $SO(10)$ (where the 10_H is needed for the electroweak symmetry breaking) fails when addressing the neutrino spectrum: in nonsupersymmetric models the $B - L$ breaking scale, M_{B-L} , turns out to be generally a few orders of magnitude below M_G [10]. Thus, the scale of the right-handed (RH) neutrino masses $M_R \sim M_{B-L}^2/M_P$ emerging first at the $d = 5$ level from an operator of the form $16_F^2(16_H^*)^2/M_P$ (with M_P typically identified with the Planck scale) undershoots by many orders of magnitude the range of about $10^{13 \div 14}$ GeV naturally suggested by the seesaw mechanism. This issue can be alleviated by considering 126_H in place of 16_H in the Higgs sector, since in such a case the neutrino masses can be generated at the renormalizable level by the term $16_F^2 126_H^*$. This lifts the problematic M_{B-L}/M_P suppression factor inherent to the $d = 5$ effective mass and yields $M_R \sim M_{B-L}$, that might be, at least in principle, acceptable [12]. Fermion masses and mixings in nonsupersymmetric $SO(10)$ with $10_H \oplus 126_H$ in the Yukawa sector have been recently reconsidered by the authors of Ref. [13], which show the potential predictivity of such a class of models.

3. Adjoint breaking in supersymmetric $SO(10)$

By invoking TeV-scale SUSY, the qualitative picture changes a lot for neutrinos. Indeed, the gauge running within the minimal supersymmetric SM (MSSM) prefers M_{B-L} in the proximity of M_G and, hence, the Planck-suppressed $d = 5$ RH neutrino mass operator $16_F^2 \overline{16}_H^2 / M_P$, available whenever $16_H \oplus \overline{16}_H$ is present in the Higgs sector, can naturally reproduce the desired scale for M_R . Let us recall that both 16_H as well as $\overline{16}_H$ are required in order to retain SUSY below the GUT scale. It is therefore very interesting to consider the minimal Higgs setting based on the lowest-dimensional representations (up to the adjoint). On the other hand, it is well known [6, 7, 8] that the relevant superpotential does not support, at the renormalizable level, a supersymmetric breaking of the $SO(10)$ gauge group to the SM. This is due to the constraints on the vacuum manifold imposed by the F - and D -flatness conditions which, apart from linking the magnitudes of the $SU(5)$ -singlet 16_H and $\overline{16}_H$ VEVs, make the the adjoint VEV $\langle 45_H \rangle$ aligned to $\langle 16_H \overline{16}_H \rangle$. As a consequence, an $SU(5)$ subgroup of the initial $SO(10)$ gauge symmetry remains unbroken.

3.1. Renormalizable vs nonrenormalizable scenarios

The alignment of the adjoint with the spinors can be broken by giving up renormalizability and allowing for effective M_P -suppressed operators in the superpotential [7]. However, this option may be rather problematic since it introduces a delicate interplay between physics at two different scales, $M_G \ll M_P$, with the consequence of spreading the GUT-scale thresholds over several orders of magnitude below M_G . In turn this may affect $d = 5$ proton decay as well as the MSSM gauge unification and it may also jeopardize the neutrino mass generation in the seesaw scheme (cf. Ref. [14] for a more detailed account of these effects). Thus, although the Planck-induced operators can provide a key to overcome the $SU(5)$ lock of the minimal SUSY $SO(10)$ Higgs model with $16_H \oplus \overline{16}_H \oplus 45_H$, such an effective scenario is prone to failure when addressing the measured proton stability and light neutrino phenomenology.

On the other hand, in the standard $SO(10)$ framework with a Higgs sector built off the lowest-dimensional representations (up to the adjoint), it is not possible to achieve a renormalizable breaking even admitting multiple copies of each type of multiplet. Firstly, with a single 45_H at play, the little group of the adjoint is $SU(5) \otimes U(1)$ regardless of the number of $16_H \oplus \overline{16}_H$ pairs. The same is true with a second 45_H added into the Higgs sector because there is no renormalizable mixing among the two 45_H 's apart from the mass term that, without loss of generality, can be taken diagonal. With a third adjoint Higgs representation at play a cubic $45_1 45_2 45_3$ interaction is allowed. However, due to the total antisymmetry of the invariant and the fact that the adjoints commute on the SM vacuum, the cubic term does not contribute to the F-term equations. From this brief excursus one concludes that the $SU(5)$ lock cannot be broken at the renormalizable level by means of representations up to the adjoint.

Remarkably, all these issues are alleviated if one considers a flipped variant of the supersymmetric $SO(10)$ unification. What we have shown in Ref. [14] is that the flipped $SO(10) \otimes U(1)_X$ scenario [15, 16, 17, 18] offers an attractive option to break the gauge symmetry down to the SM (and further to $SU(3)_c \otimes U(1)_Q$) at the renormalizable level and by means of a quite simple Higgs sector, namely a couple of $SO(10)$ spinors $16_{H_{1,2}} \oplus \overline{16}_{H_{1,2}}$ and one adjoint 45_H .

3.2. Hypercharge embeddings in $SO(10) \otimes U(1)_X$

The so called flipped realization of the $SO(10)$ gauge symmetry requires an additional $U(1)_X$ gauge factor in order to provide an extra degree of freedom for the SM hypercharge identification. For a fixed embedding of the $SU(3)_c \otimes SU(2)_L$ subgroup within $SO(10)$, the SM hypercharge can be generally spanned over the three remaining Cartans generating the abelian $U(1)^3$ subgroup of the $SO(10) \otimes U(1)_X / (SU(3)_c \otimes SU(2)_L)$ coset. There are two consistent implementations

of the SM hypercharge within the $SO(10)$ algebra (commonly denoted by standard and flipped $SU(5)$), while a third one becomes available due to the presence of $U(1)_X$.

In order to discuss the different embeddings we adopt the traditional left-right (LR) basis corresponding to the $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ subalgebra of $SO(10)$. In full generality one can span the SM hypercharge on the generators of $U(1)_R \otimes U(1)_{B-L} \otimes U(1)_X$:

$$Y = \alpha T_R^{(3)} + \beta(B - L) + \gamma X. \quad (17)$$

The $U(1)_X$ charge is been conveniently fixed to $X_{16} = +1$ for the spinorial representation and thus $X_{10} = -2$ and also $X_1 = +4$ for the $SO(10)$ vector and singlet, respectively; this is also the minimal way to obtain an anomaly-free $U(1)_X$, that allows $SO(10) \otimes U(1)_X$ to be naturally embedded into E_6 . It is a straightforward exercise to show that there are only three solutions which accommodate the SM matter quantum numbers over a reducible $16 \oplus 10 \oplus 1$ representation. On the $U(1)_R \otimes U(1)_{B-L} \otimes U(1)_X$ bases of Eq. (17) one obtains,

$$\alpha = 1, \beta = \frac{1}{2}, \gamma = 0, \quad (18)$$

which is nothing but the “standard” embedding of the SM matter into $SO(10)$. The second option is characterized by

$$\alpha = -1, \beta = \frac{1}{2}, \gamma = 0, \quad (19)$$

which is usually denoted “flipped $SU(5)$ ” [19, 20] embedding and corresponds to a sign flip of the $SU(2)_R$ Cartan operator $T_R^{(3)}$. A third solution corresponds to

$$\alpha = 0, \beta = -\frac{1}{4}, \gamma = \frac{1}{4}, \quad (20)$$

denoted as “flipped $SO(10)$ ” [15, 16, 17, 18] embedding of the SM hypercharge. Notice, in particular, the fundamental difference between the setting (20) with $\gamma = \frac{1}{4}$ and the two previous cases (18) and (19) where $U(1)_X$ does not play any role.

3.3. The supersymmetric flipped $SO(10) \otimes U(1)_X$ model

The active role of the $U(1)_X$ generator in the SM hypercharge identification within the flipped $SO(10)$ scenario has relevant consequences for model building. In particular, the SM decomposition of the $SO(10)$ representations changes so that there are additional SM-singlets both in $16_H \oplus \overline{16}_H$ as well as in 45_H . The presence of these additional SM-singlets provides the ground for obtaining a viable symmetry breaking with a significantly simplified renormalizable Higgs sector.

Naively, one may guess that the pair of VEVs in 16_H (plus another conjugated pair in $\overline{16}_H$ to maintain the required D -flatness) might be enough to break the GUT symmetry entirely, since one component transforms as a 10 of $SU(5) \subset SO(10)$ (cf. s_H in Table 1), while the other one (cf. n_H in Table 1) is identified with an $SU(5)$ singlet. Nevertheless, flipping is not per-se sufficient since the adjoint does not reduce the rank and the bi-spinor, in spite of the two qualitatively different SM-singlets involved, can lower it only by a single unit, leaving a residual $SU(5) \otimes U(1)$ symmetry. Only when two pairs of $16_H \oplus \overline{16}_H$ (interacting via 45_H) are introduced the two pairs of SM-singlet VEVs in the spinor multiplets may not generally be aligned and the little group is reduced to the SM [14]. Given the most general renormalizable Higgs superpotential, made of the representations $45_H \oplus 16_{H_1} \oplus \overline{16}_{H_1} \oplus 16_{H_2} \oplus \overline{16}_{H_2}$

$$W_H = \frac{\mu}{2} \text{Tr} 45_H^2 + \rho_{ij} 16_{H_i} \overline{16}_{H_j} + \tau_{ij} 16_{H_i} 45_H \overline{16}_{H_j}, \quad (21)$$

where $i, j = 1, 2$, the study of the SUSY vacuum in Ref. [14] shows that the little group is the SM for large portions of the parameter space in which the VEVs of the $16_H \oplus \overline{16}_H$ pairs are not aligned.

	$SO(10)$	$SO(10)_f$
16_F	$(D^c \oplus L)_{\bar{5}} \oplus (U^c \oplus Q \oplus E^c)_{10} \oplus (N^c)_1$	$(D^c \oplus \Lambda^c)_{\bar{5}} \oplus (\Delta^c \oplus Q \oplus S)_{10} \oplus (N^c)_1$
10_F	$(\Delta \oplus \Lambda^c)_{\bar{5}} \oplus (\Delta^c \oplus \Lambda)_{\bar{5}}$	$(\Delta \oplus L)_{\bar{5}} \oplus (U^c \oplus \Lambda)_{\bar{5}}$
1_F	$(S)_1$	$(E^c)_1$
$\langle 16_H \rangle$	$(0 \oplus \langle H_d \rangle)_{\bar{5}} \oplus (0 \oplus 0 \oplus 0)_{10} \oplus (n_H)_1$	$(0 \oplus \langle H_u \rangle)_{\bar{5}} \oplus (0 \oplus 0 \oplus s_H)_{10} \oplus (n_H)_1$
$\langle \overline{16}_H \rangle$	$(0 \oplus \langle H_u \rangle)_{\bar{5}} \oplus (0 \oplus 0 \oplus 0)_{\overline{10}} \oplus (n_H)_1$	$(0 \oplus \langle H_d \rangle)_{\bar{5}} \oplus (0 \oplus 0 \oplus s_H)_{\overline{10}} \oplus (n_H)_1$

Table 1. SM decomposition of $SO(10)$ representations in the standard (left) and flipped (right) hypercharge embedding. A self-explanatory SM notation is used, with the outer subscripts labeling the $SU(5)$ origin.

Let us stress that in the flipped embedding the spinor representations include also weak doublets H_u and H_d that may trigger the electroweak symmetry breaking and allow for renormalizable Yukawa interactions with the chiral matter fields distributed in the flipped embedding over a reducible $16_F \oplus 10_F \oplus 1_F$ representation. Notice that this matter content is needed in order to cancel the gauge anomalies of the $U(1)_X$ factor and to correctly reproduce the SM matter quantum numbers (cf. Table 1).

Considering for simplicity just one pair of spinor Higgs multiplets and imposing a Z_2 matter-parity (negative for matter and positive for Higgs superfields) the most general Yukawa superpotential (up to $d = 5$ operators) reads

$$W_Y = Y_U 16_F 10_F 16_H + \frac{1}{M_P} [Y_E 10_F 1_F \overline{16}_H \overline{16}_H + Y_D 16_F 16_F \overline{16}_H \overline{16}_H] , \quad (22)$$

where family indexes are understood. Notice (cf. Table 1) that due to the flipped embedding the up-quarks receive mass at the renormalizable level, while all the other fermion masses need Planck-suppressed effective contributions in order to achieve a realistic texture. Thus the top/bottom hierarchy is given by an $M_G/M_P \sim 10^{-2}$ factor, which selects naturally $\mathcal{O}(1)$ values for $\tan \beta \equiv v_u/v_d$. At the end, it can be shown [14] that the Yukawa superpotential in Eq. (22) can reproduce realistic textures for the SM fermions (including neutrinos), while the exotic states are automatically kept heavy by the symmetry breaking pattern.

3.4. Minimal E_6 embedding

The mechanism we advocate can be embedded in an underlying nonrenormalizable E_6 Higgs model featuring a pair of $27_H \oplus \overline{27}_H$ and the adjoint 78_H . Technical similarities apart, there is, however, a crucial difference between the $SO(10) \otimes U(1)_X$ and E_6 scenarios, that is related to the fact that the Lie-algebra of E_6 is larger than that of $SO(10) \otimes U(1)_X$. It has been shown long ago [21] that the renormalizable SUSY E_6 Higgs model spanned over $27_H \oplus \overline{27}_H \oplus 78_H$ leaves an $SO(10)$ symmetry unbroken. Two pairs of $27_H \oplus \overline{27}_H$ are needed to reduce the rank by two units. In spite of the fact that the two SM-singlet directions in the 27_H are exactly those of the “flipped” 16_H , the little group of the $2 \times (27_H \oplus \overline{27}_H) \oplus 78_H$ Higgs sector remains at the renormalizable level $SU(5)$, as we explicitly show in Ref. [14]. Adding nonrenormalizable adjoint interactions allows for a misalignment of the $\langle 78_H \rangle$ from the $SU(5) \otimes U(1) \otimes U(1)$ direction, such that the little group is reduced to the SM. Since a one-step E_6 breaking with nonrenormalizable operators is phenomenologically problematic as mentioned earlier, we argue for a two-step breaking, via flipped $SO(10) \otimes U(1)_X$, with the E_6 scale near the Planck scale. Barring detailed threshold effects, it is interesting to see from Fig. 1 that the few percent mismatch observed within the two-loop MSSM gauge coupling evolution at the scale of the “one-step” grand unification is naturally accommodated in this scheme, and it is understood as an artefact of a “delayed” E_6 unification superseding the flipped $SO(10) \otimes U(1)_X$ partial unification.

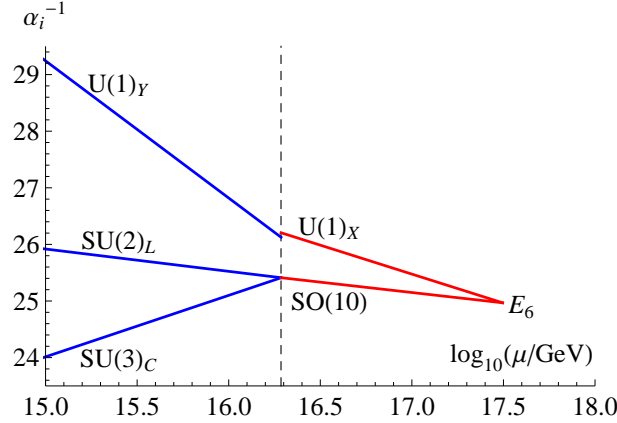


Figure 1. Sample picture of gauge unification in the E_6 -embedded $SO(10) \otimes U(1)_X$ model.

4. Conclusions

Longstanding results claimed that nonsupersymmetric $SO(10)$ GUTs with just the adjoint triggering the first stage of the GUT breaking cannot provide a successful gauge unification. We argued that this conclusion is an artefact of the tree level potential and showed that quantum corrections have a dramatic impact. In particular, a model featuring $10_H \oplus 126_H \oplus 45_H$ in the Higgs sector has all the ingredients to be a viable minimal nonsupersymmetric $SO(10)$ GUT candidate [12]. Analogously, supersymmetric $SO(10)$ GUTs with representations up to the adjoint do not provide a phenomenologically viable breaking to the SM. We pointed out that the flipped $SO(10)$ embedding offers an attractive setting for breaking the gauge symmetry directly to $SU(3)_c \otimes U(1)_Q$ at the renormalizable level, by means of a quite simple Higgs sector: $2 \times (16_H \oplus \overline{16}_H) \oplus 45_H$. The case is made for a two-step breaking of a supersymmetric E_6 GUT realised in the vicinity of the Planck scale via an intermediate flipped $SO(10) \otimes U(1)_X$ stage.

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